STAT 2593

Lecture 038 - The Simple Linear Regression Model

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The Simple Linear Regression Model

Learning Objectives

1. Describe the simple linear regression model and the constituent components.

2. Understand the normal assumptions for the simple linear regression model.

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  - The effect of treatment on a health outcome.
  - Housing factors influencing the cost of a home.
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- When we are interested in describing this relationship directly, we typically use **regression models**.

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  - β_0 is the intercept for the line.
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- The **simple linear regression model** takes this, and makes it probabilistic.

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  - The randomness comes from ϵ.

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- If we have values for x, and estimates of β_0 and β_1, then we can predict values of Y.
  - If we assume normality, we can also predict intervals around these predictions.

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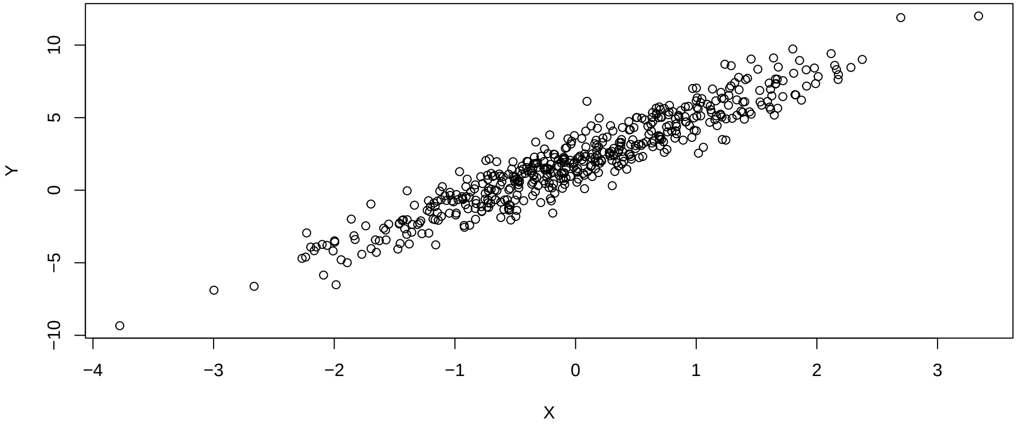
- Scatterplots are useful for determining the relationship between two different variables, and in particular, assessing whether a specified relationship looks reasonable.
- In our case: does it seem like a straight line would fit the data well?

Examples



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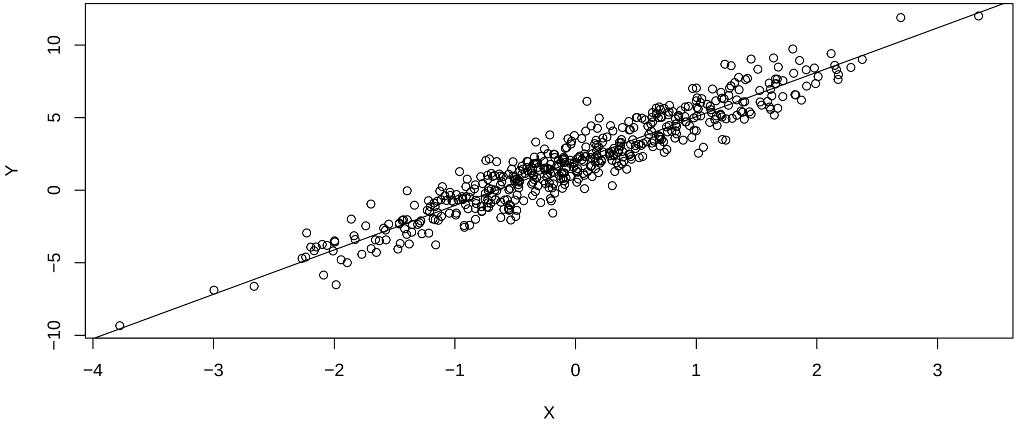

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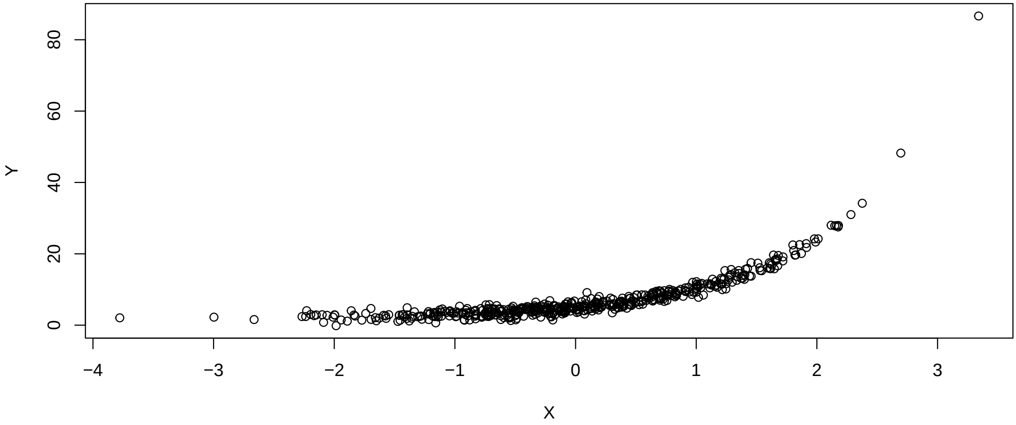

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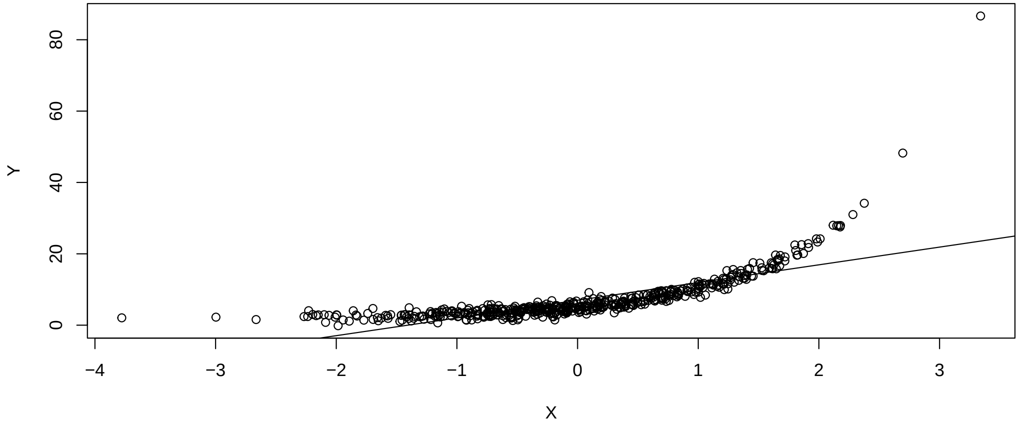

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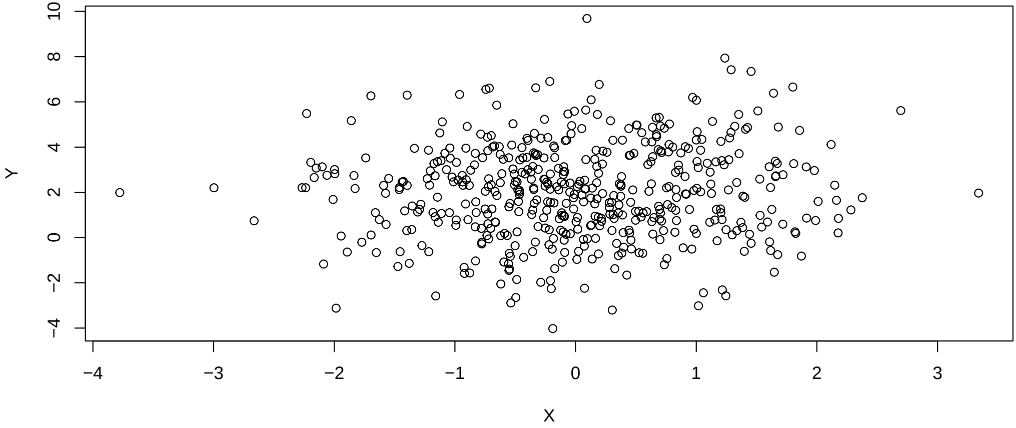

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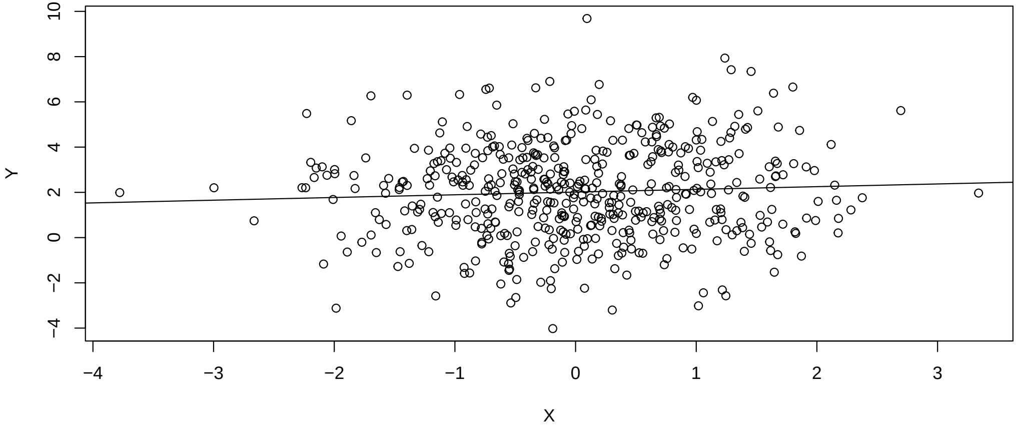

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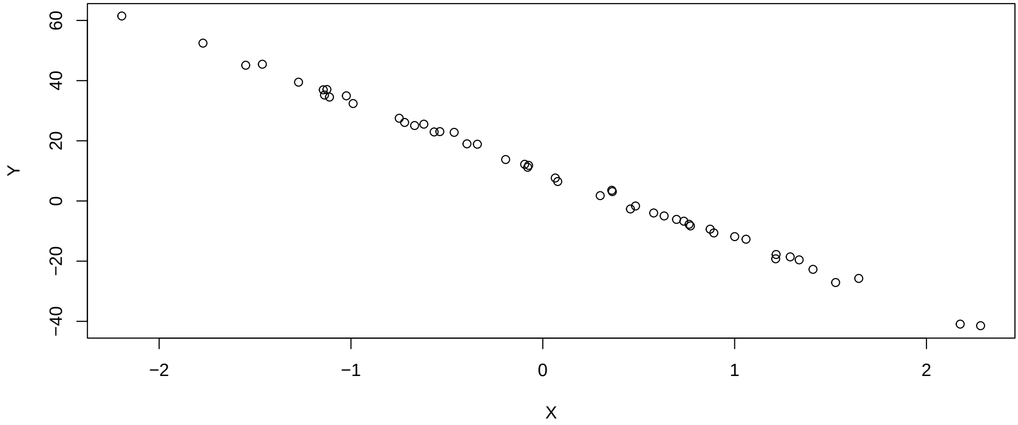

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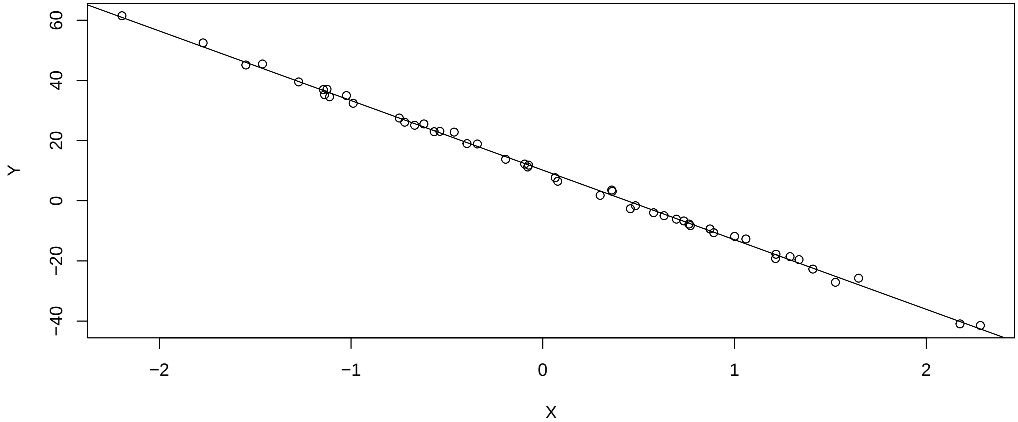

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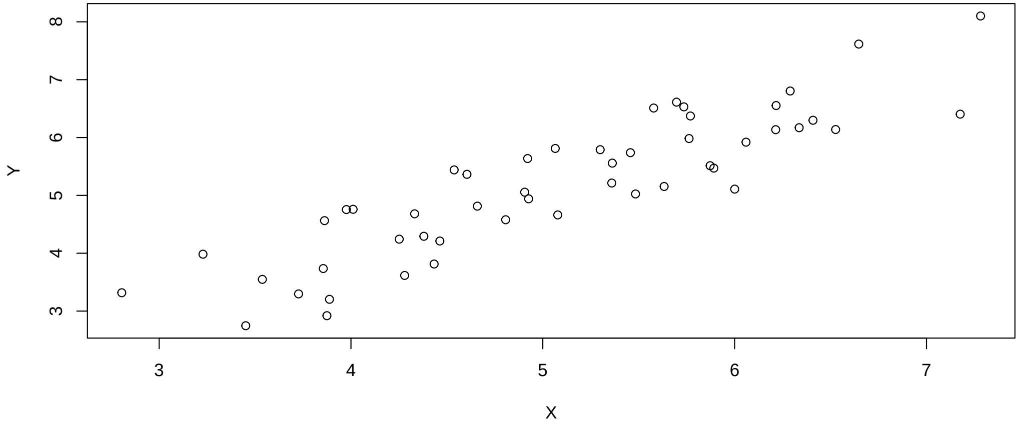

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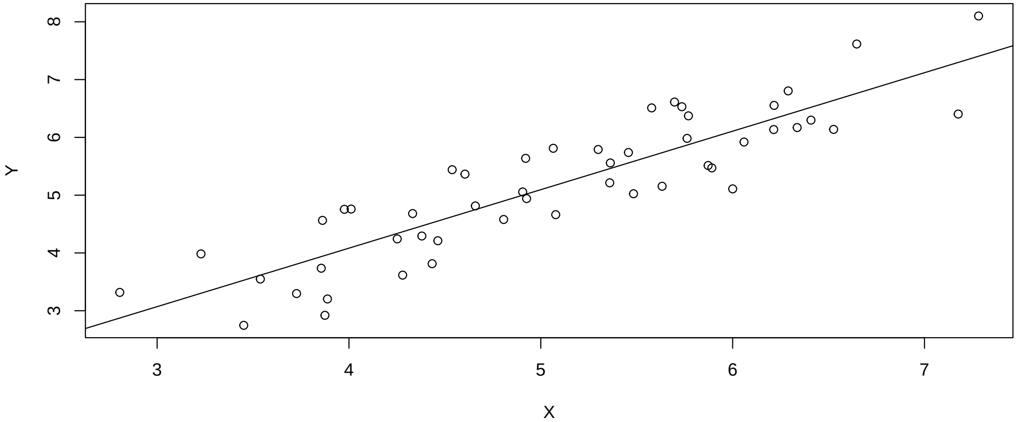

Summary

- Linear relationships are the simplest relationships to capture dependency between two variables.
- The linear regression model adds randomness through the use of an error term.
- We can use scatterplots to assess the linearity of a relationship (and perhaps to find transformations that make it more linear!).